A Unified Modal Analysis of Off-Centered Waveguide Junctions With Thick Iris

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***Abstract*—In this paper, the unified mode-matching technique for concentric waveguide junction analysis is extended to calculate the scattering parameters at off-centered step discontinuities in waveguides. Because the modal fields in both waveguides are expressed accurately by polynomials in mono coordinate system, the analytical expressions of the coupling coefficients are easily obtained. Furthermore, all waveguides are tacked with in a unified manner. Comparison with the data available in literature demonstrates the accuracy and flexibility of the present method.**

***Index Terms*—Mode-matching method, off-centered waveguide junction, unified analysis.**

I. Introduction

A unified and efficient mode-matching technique has been proposed for the analysis of concentric waveguide junctions, where both waveguides can be rectangular, circular, elliptical, or their different combinations [1]. This technique is based on the unified polynomial approximation method of waveguide modal analysis [2]–[4], where the higher-order modes in regular waveguides can be determined accurately by the QZ factorization for generalized eigenvalue problems [5]. The investigation was limited to simple cases of two waveguides concentrically connected. The rigorous and efficient analysis of off-centered waveguide junctions is often required in characterizing more complicated structures, such as filters and multiplexing networks. This paper presents the extension of this unified and efficient mode-matching technique to off-centered cases.

II. Scattering Matrix Formulation of Waveguide Junction

The off-centered waveguide junction is depicted in Fig. 1(a), and the geometry of thick off-centered iris in waveguide, which can be seen as two cascading junctions, is drawn in Fig. 1(b). The thickness of the iris is denoted as t. The larger waveguide, represented by parameters a_1, b_1, γ_1, with Cartesian coordinates (x_1, y_1, z_1), is transversely offset from the smaller waveguide whose parameters are a_2, b_2, γ_2 with Cartesian coordinates (x_2, y_2, z_2) such that

$$x_1 = x_2 + \Delta x \quad (1a)$$

$$y_1 = y_2 + \Delta y \quad (1b)$$

$$z_1 = z_2. \quad (1c)$$

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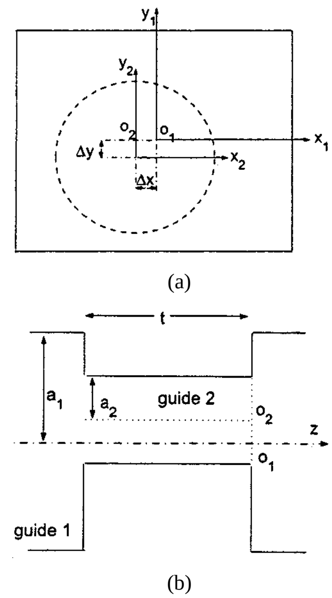


Fig. 1. Cross section and side views of structures under study [(a) off-centered waveguide junction and (b) thick iris in waveguide].

The longitudinal fields, e_z of TM modes (e-type) and h_z of TE modes (h-type) in a hollow conducting waveguide, are solved by the unified method in [2], [3] and expressed as follows

$$e_z(x,y) = \sum_{i=1}^{m} C_i \Psi(x,y) x^r y^s \quad (2)$$

$$h_z(x,y) = \sum_{j=1}^{n} C_j x^r y^s \quad (3)$$

where the definitions of function $\Psi(x,y)$, r, and s have been given in [6].

By using QZ factorization for generalized eigenvalue problems, both the lower-order and the higher-order modes in regular waveguides have been determined accurately [5]. For example, the first 100 successive modes in an elliptical waveguide with arbitrary eccentricity have been obtained and validated by results from the analytical method. Once the longitudinal fields are known, the transverse modal fields $\mathbf{e^h}$ (TE mode) and $\mathbf{e^e}$ (TM mode) can be derived from the Maxwell equations.

By following the standard mode-matching procedure in [7], the scattering matrix of waveguide junction is obtained after some manipulations

$$[S_{22}] = \left([Y_2] + [M]^T [Y_1] [M]\right)^{-1} \times \left([Y_2] - [M]^T [Y_1] [M]\right) \quad (4)$$

$$[S_{21}] = 2\left([Y_2] + [M]^T [Y_1] [M]\right) [M]^T [Y_1] \tag{5}$$

$$[S_{12}] = [M] ([S_{22}] + [I]) \tag{6}$$

$$[S_{11}] = [M] [S_{21}] - [I] \tag{7}$$

where

$$[M] = \begin{bmatrix} [H] & [K] \\ [Q] & [E] \end{bmatrix} \tag{8}$$

Y_i modal admittance matrix of the ith waveguide with $i = 1, 2$;
$[I]$ identity matrix;
T transpose operation.

Taken as two cascading waveguide junctions, the thick iris in waveguides can be studied by the generalized scattering matrix technique, where the formulation of the scattering matrix has already been given in [7].

In (8), $[H]$, $[K]$, $[Q]$, and $[E]$ represent the TE-TE, TE-TM, TM-TE and TM-TM E-field mode-coupling coefficients and they are evaluated respectively by the following integrals

$$H_{mn} = \int_{S_2} \mathbf{e}^{\mathbf{h}}_{(\mathbf{1})\mathbf{m}} \cdot \mathbf{e}^{\mathbf{h}}_{(\mathbf{2})\mathbf{n}} \mathbf{dS} \tag{9}$$

$$K_{mn} = \int_{S_2} \mathbf{e}^{\mathbf{h}}_{(\mathbf{1})\mathbf{m}} \cdot \mathbf{e}^{\mathbf{e}}_{(\mathbf{2})\mathbf{n}} \mathbf{dS} \tag{10}$$

$$Q_{mn} = \int_{S_2} \mathbf{e}^{\mathbf{e}}_{(\mathbf{1})\mathbf{m}} \cdot \mathbf{e}^{\mathbf{h}}_{(\mathbf{2})\mathbf{n}} \mathbf{dS} \tag{11}$$

$$E_{mn} = \int_{S_2} \mathbf{e}^{\mathbf{e}}_{(\mathbf{1})\mathbf{m}} \cdot \mathbf{e}^{\mathbf{e}}_{(\mathbf{2})\mathbf{n}} \mathbf{dS} \tag{12}$$

where S_2 denotes the cross section of the smaller waveguide.

Evaluating the integrals in (9)–(12) accurately and efficiently is the key step in efficient CAD of waveguide junctions. In literature, analytical expressions of above integrals are derived usually case by case and one–dimensional (1-D) numerical integral is still needed sometimes. In [1], we have proposed a unified mode-matching method to analyze concentric waveguide junctions, where the unified analytical expressions of all cases are used. Because the integrals is over S_2, the transverse modal fields in the larger waveguide need to be expressed in coordinates of the smaller waveguide. This task becomes very easy here, because only polynomials in Cartesian coordinate system are involved in both waveguide modal field expressions in (2) and (3), and no special functions or other coordinates are used. With (1a), (1b) and the following binomial theorem [8]

$$x_1^n = (x_2 + \Delta x)^n = \sum_{i=0}^{n} C_n^i \, x_2^i \, \Delta x^{n-i} \tag{13a}$$

$$y_1^m = (y_2 + \Delta y)^m = \sum_{i=0}^{m} C_m^i \, y_2^i \, \Delta y^{m-i} \tag{13b}$$

where

$$C_n^i = \frac{n!}{i!(n-i)!} \tag{14}$$

the modal fields in larger waveguide are transformed to coordinates (x_2, y_2, z_2). Now, field expressions in both waveguides are in the same coordinate system (x_2, y_2, z_2) and the analytical expressions are available [1].

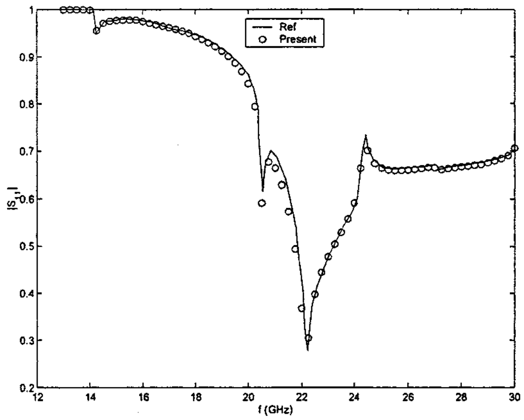
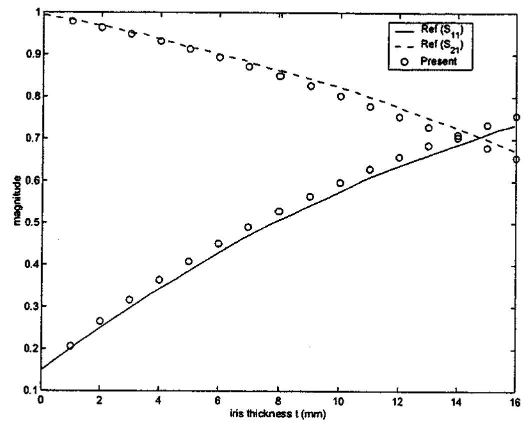
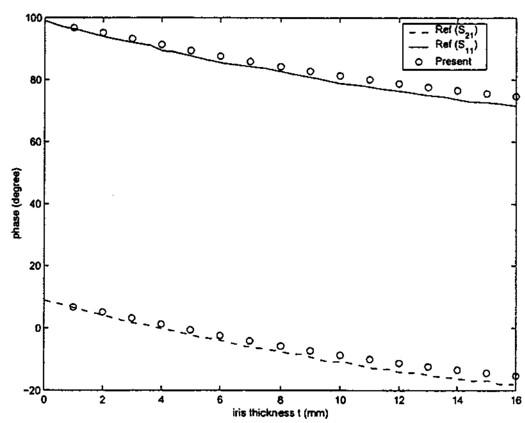


Fig. 2. Comparison of reflection coefficients against frequency obtained from the present method and the results in [9] ($a_1 = 1.0$ cm, $b_1 = 0.7$ cm, $a_2 = 0.4$ cm, $b_2 = 0.3$ cm, $\Delta x = 0.3$ cm, and $\Delta y = 0.2$ cm).



(a)



(b)

Fig. 3. Magnitudes and phases of scattering coefficients versus iris thickness from the present method and reference [11] ($a_1 = b_1 = 12.7$ mm, $a_2 = b_2 = 9.5$ mm, $\Delta x = 1.5$ mm, $\Delta y = 0$, and $f = 9$ GHz).

III. Numerical Examples

In this section, scattering parameters at several off-centered waveguide junctions are calculated and compared with the results of other approaches to show the accuracy of the present

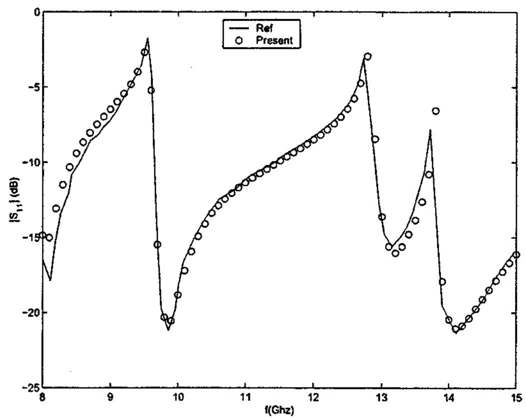


Fig. 4. Reflection coefficients in dB of the circular-to-rectangular waveguide junction with offset from the present method and reference [13] ($a_1 = 0.375"$, $b_1 = 0.1785"$, $a_2 = b_2 = 0.75"$, $\Delta x = -0.125"$, and $\Delta y = 0.001\,25"$).

method. The first example is the off-centered elliptical waveguide junction. In [9], trigonometric and Bessel function expansions were used to express the modal fields in elliptical waveguides and the 1-D contour integrals, which were transformed from surface integrals in (9)–(12) using first Green's identity, were numerically evaluated. The parameters of the waveguide junction are $a_1 = 1.0$ cm, $b_1 = 0.7$ cm, $a_2 = 0.4$ cm, $b_2 = 0.3$ cm, $\Delta x = 0.3$ cm, and $\Delta y = 0.2$ cm. The comparison of the reflection coefficients as a function of frequency is given in Fig. 2. From the figure, it can be seen that two results agree very well in the whole frequency range.

The second example is the thick off-centered circular iris in circular waveguides. This structure has been investigated in [10] and [11]. In [10], the conservation of complex power technique (CCPT) was used to derive the analytical solutions and the cutoff iris-waveguide assumption was made. In [11], the least-squares boundary residual method (LSBRM) was applied to eliminate above assumption, thus the operating frequency range was extended. Like in [11], there is no such an assumption in the present method. The radius of the circular waveguide is $a_1 = b_1 = 12.7$ mm and the radius of the circular iris is $a_2 = b_2 = 9.5$ mm. The eccentric distance are: $\Delta x = 1.5$ mm and $\Delta y = 0$. The magnitudes and phases of reflection and transmission coefficients are plotted as a function of iris thickness in Figs. 3(a) and (b), respectively. For magnitude and phase, the results predicated by the present method agree favorably with those from LSBRM.

The last example is the off-centered circular-to-rectangular waveguide junction. This problem has been treated in [12] and [13]. The cylindrical Bessel–Fourier modal fields of the circular waveguide were transformed into a finite series of exponential plane wave functions in [13] to obtain the analytical finite series solutions of the coupling coefficients. The rectangular waveguide is WR75 with $a_1 = 0.375"$ and $b_1 = 0.1785"$, the radius of the circular waveguide is $a_2 = b_2 = 0.75"$, and the offset distances are $\Delta x = -0.125"$ and $\Delta y = 0.001\,25"$. The behavior of $|S_{11}|$ in dB is given in Fig. 4. Again, a good agreement is observed. In the all three examples, the number of polynomials used is 150.

IV. Conclusion

In this paper, using the binomial theorem, the unified and efficient mode-matching method in [1] is extended to off-centered waveguide junctions. Comparisons with results from other methods show the accuracy of and make a further validation to the present method. The unified analytical expressions of coupling coefficients make the present method high numerical efficient in the rigorous analysis of the investigated junctions. Besides the advantages of accuracy, efficiency, and flexibility, which have been shown in [1], the capability of straightforward extension to nonconcentric cases can be considered as another advantage of the present method.

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